

AN INVESTIGATION OF THE VIRTUAL MASS  
OF A CYLINDER VIBRATING IN WATER

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DAVID A. ROGERS  
AND  
MACLEAN C. SHAKSHOBER  
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An Investigation of the Virtual Mass

of a

Cylinder Vibrating in Water

by

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Submitted to the Department of Naval Architecture and Marine Engineering on  
May 25, 1953 in partial fulfillment of the requirements for the degree of  
Naval Engineer.

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## ABSTRACT

### AN INVESTIGATION OF THE VIRTUAL MASS OF A CYLINDER VIBRATING IN WATER

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Submitted to the Department of Naval Architecture and Marine Engineering on May 25, 1953 in partial fulfillment of the requirements for the degree of Naval Engineer.

The object of this thesis is to experimentally investigate the virtual mass of a hollow cylinder vibrating in water.

A lucite cylinder was magnetically vibrated in air and water at various length to diameter ratios and the frequency of vibration for as many modes as possible, up to five, recorded. No attempt was made to measure amplitude. The ratio of added water mass to displaced water mass was computed from the frequencies and compared with analytical results.

The investigation shows that end effects have a very great influence on virtual mass. As the length to diameter ratio is decreased, the added virtual water mass is decreased. There is also a decrease in virtual water mass as frequency is increased at constant length to diameter ratios.

A ratio of the measured virtual water mass to analytical, called K, was computed and found to be a function of  $L/D$  and mode number.

It is recommended that further investigations using bodies of revolution whose ends have zero area such as ellipsoids be made. It would be desirable to use equipment to permit measuring amplitude as well as frequency.

Where data is available, it is recommended that an attempt to calculate the frequencies of an actual hull such as a submarine be made, correcting the analytical virtual water mass by the applicable K values.

Thesis Supervisor: Professor F. M. Lewis  
Title: Professor of Marine Engineering

EXPERIMENTAL INVESTIGATION OF THE EFFECT OF VIBRATION  
ON THE STABILITY OF A SHIP

by

WILLIAM C. SHAW, JR.  
LIEUTENANT, U.S. NAVY

and

DAVID A. ROBERTS  
LIEUTENANT, U.S. NAVY

Submitted to the Department of Naval Architecture and Marine Engineering on  
May 22, 1953 in partial fulfillment of the requirements for the degree of  
Naval Engineer.

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A Lucite cylinder was experimentally vibrated in air and water at various  
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mass as frequency is increased at constant length to diameter ratios.

A ratio of the measured virtual water mass to analytical, called  $k$ , was  
computed and found to be a function of  $L/D$  and mode number.

It is recommended that further investigation using bodies of revolution  
whose ends have zero area and an elliptical be made. It would be desirable  
to use equipment to permit measuring amplitude as well as frequency.

Where data is available, it is recommended that an attempt be made to correlate  
the frequencies of an actual hull such as a submarine be made, correcting the  
analytical virtual water mass by the applicable  $k$  values.

WILLIAM C. SHAW, JR.  
LIEUTENANT, U.S. NAVY

Cambridge, Massachusetts  
May 25, 1953

Professor Earl B. Millard  
Secretary of the Faculty  
Massachusetts Institute of Technology  
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the degree of Naval Engineer,  
we herewith submit a thesis entitled "An Investigation of the Virtual Mass  
of a Cylinder Vibrating in Water."

Respectfully,

---

David A. Rogers  
Lieutenant, U. S. Navy

---

MacLean C. Shakshober  
Lieutenant, U. S. Navy

Cambridge, Massachusetts  
May 22, 1955

Professor Earl R. Hilgard  
Secretary of the Faculty  
Massachusetts Institute of Technology  
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the degree of Naval Engineer,  
we herewith submit a thesis entitled "An Investigation of the Virtual Mass  
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Respectfully,

David A. Rogers  
Lieutenant, U. S. Navy

Professor C. S. Shull  
Lieutenant, U. S. Navy

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## I. INTRODUCTION

When immersed in a dense fluid, such as water, a body vibrates as though it had undergone an increase in mass. This increase in mass is due to the flow of fluid about the body as it moves.

Several analytical methods of computing the virtual mass of a body vibrating in a dense fluid have been presented.<sup>(1, 2)</sup> These methods are based on the assumption of potential flow about the body. Unless the body is of uniform shape with pointed ends, i.e. an ellipsoid, it is not presently possible to compute analytically the virtual mass because of the flow about the ends being highly rotational.

A submerged submarine must vibrate as a free-free body at a frequency determined by its virtual mass as described above. When vibrating horizontally, its virtual mass will be somewhat lower than that computed on the basis of no end losses because of flow about the ends.

Professor Frank M. Lewis has presented a method for determining the virtual mass based on an ellipsoid which may be corrected for other shapes.<sup>(1)</sup> Dr. H. M. Schauer of the Underwater Explosion Research Division, Norfolk Naval Shipyard, has done likewise for a cylinder with no end flow. Mr. E. B. Moullin and Mr. A. D. Browne, in a paper presented before the Cambridge Philosophical Society in 1928<sup>(3)</sup> gave the results of their investigation of the periods of a free-free bar of rectangular cross section vibrating in water. In their experiments they used long bars which had length to depth ratios of from 26 to 39. Using such long bars they found that flow about the ends did not have any appreciable effect on the virtual mass as analytically computed. However, they did not investigate lower length to depth ratios. They found that the virtual mass is not affected by depth when below about two diameters.

# APPENDIX

When immersed in a fluid, a body is subjected to a buoyant force which is equal to the weight of the fluid displaced. This force is due to the pressure of the fluid on the body, and it acts through the center of buoyancy, which is the center of gravity of the displaced fluid. The center of buoyancy is not necessarily the same as the center of gravity of the body. If the center of buoyancy is above the center of gravity, the body is stable; if it is below, the body is unstable; if it is on the same point, the body is neutrally stable.

Several analytical methods of computing the virtual mass of a body vibrating in a dense fluid have been presented. (1) (2) These methods are based on the assumption of potential flow about the body. Unless the body is of uniform shape with pointed ends, i.e., an ellipsoid, it is not presently possible to compute analytically the virtual mass because of the flow about the ends being highly rotational.

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The following is a report on the experimental determination of the virtual mass of a circular cylinder for several length to diameter ratios while vibrating in water.

and the following table shows the results of the  
 various methods of determining the relative  
 values of the different elements.

## II. PROCEDURE

In this section the steps followed to obtain the desired results are described. The procedure consists of two parts, experimental and analytical. Details of the procedure are presented in the Appendix.

### Experimental Procedure

From "The Theory of Sound" by Rayleigh<sup>(4)</sup> the appropriate equations were used to obtain the nodal and anti-nodal points of a free-free bar. Using the frequency equation for a free-free bar

$$f = \frac{n^2}{2\pi} \left[ \frac{EK^2}{\delta} \right]^{1/2} \quad (1)$$

the first five modes were computed to give an approximation of the natural frequencies.

To vibrate the cylinder mechanically would require a motor with a speed range of 1800 to 60,000 revolutions per minute. For this reason, magnetic vibration of the bar was by far the preferred method. Schematics of the apparatus are shown in Figure I.

A Lucite plastic tube 52.7 inches long with an outer diameter of 2 inches and an inner diameter of 1.75 inches was used for the first test. On the end of the cylinder was wrapped some small diameter soft iron wire to permit magnetic excitation. The amount of wire was not great enough to affect the frequency or mass of the bar. By experimenting with various types of pickups, it was found that a seismic crystal gave the best results. The pickup was very light in weight and very sensitive to vibration. This particular pickup was a Brush seismic crystal used on the sounding board of an electric guitar. The pickup was mounted on the inside of the cylinder at the opposite end of

2. Results

The first series of experiments was carried out with a constant frequency of 1000 cycles per second. The results are shown in Figure 1.

Experimental Procedure

From the theory of the experiment it was expected that the results would be independent of the frequency of the sound waves. To test this, experiments were carried out at frequencies of 500, 1000, and 1500 cycles per second. The results are shown in Figure 2.



(1)

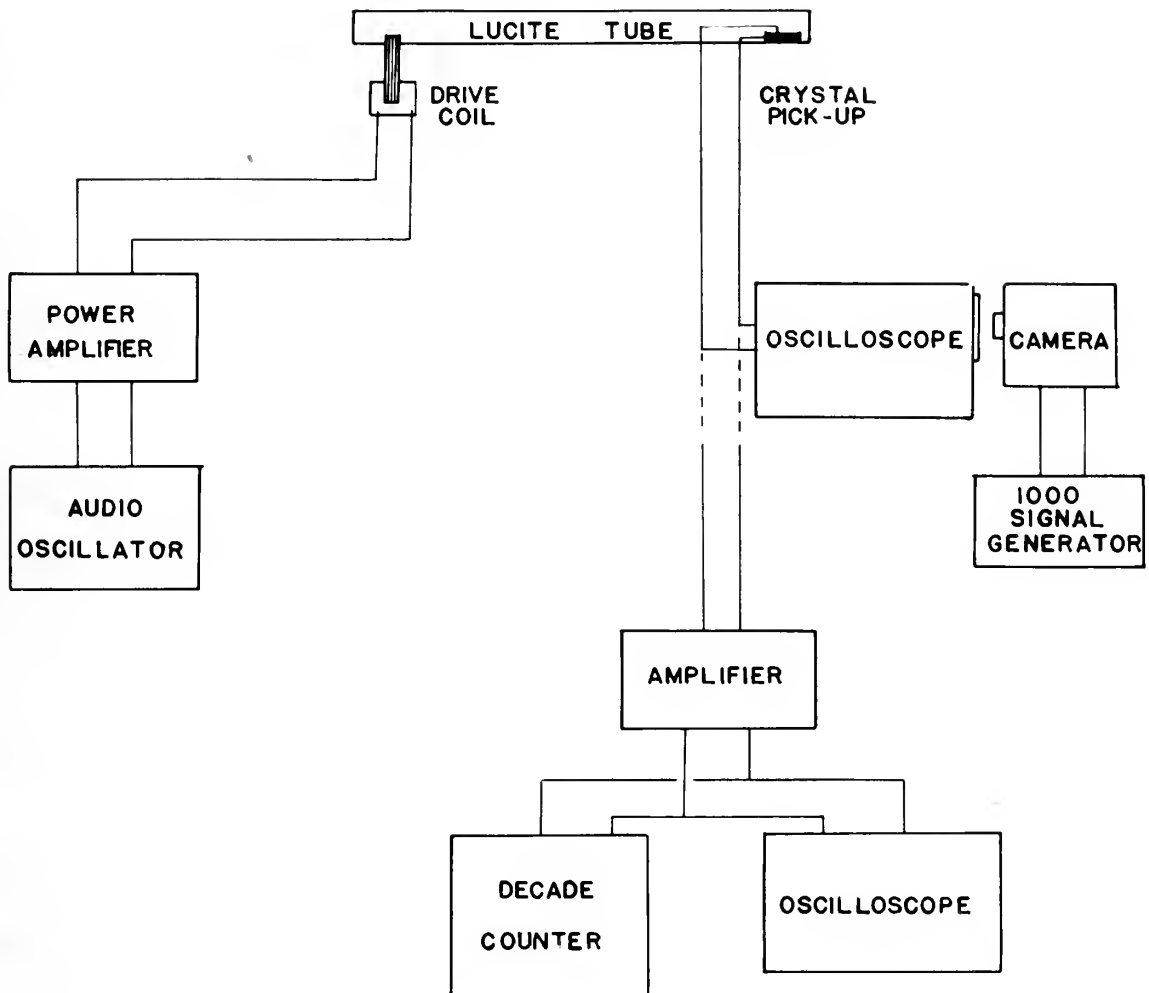
The first series of experiments was carried out with a constant frequency of 1000 cycles per second. The results are shown in Figure 1.

To find the critical frequency of the system, experiments were carried out at frequencies of 500, 1000, and 1500 cycles per second. The results are shown in Figure 2.

A further series of experiments was carried out with a constant frequency of 1000 cycles per second. The results are shown in Figure 3. The amplitude of the oscillations was found to be independent of the frequency of the sound waves. This is in agreement with the theory of the experiment.

FIG. I  
SCHEMATIC

MAY 8, 1953, MCS







the exciting wire. A small hole was drilled at a nodal point through which the wires from the pickup were run.

An audio oscillator with a frequency range of 20 to 20,000 cycles per second was used to drive an electro-magnet. The output of the audio oscillator was amplified in a power amplifier and this was used to drive the electro-magnet. The output of the crystal pickup was put into a cathode ray oscilloscope where the signal was peaked for resonance. Unfortunately, due to radiation, frequencies above 1600 cycles per second could not be detected.

Two different types of frequency measurement were employed to compute the frequency of vibration, both of which gave very accurate and like results. The first method was to take the output of the pickup and put it on the vertical plates of the CRO. When a resonant signal was obtained, a picture of the frequency was taken with no horizontal sweep. The camera used was a very high speed model with no shutter. The camera had a built-in timing light of 1000 cycles per second which showed on the film. The frequency was then computed from the developed film by counting the cycles.

The other method of determining the frequency was to take the amplified output of the crystal pickup and put it into an electronic decade counter.

To compute the air frequencies, the cylinder was suspended by strings located at the nodal points. The electro-magnet was placed as close as possible to the soft iron windings. To prevent banging of the cylinder, a rubber band was used as a standoff. The frequencies were recorded as described above.

For the water tests, the cylinder was immersed seven diameters in the towing tank in the M. I. T. Hydrodynamics Laboratory. This depth ensured that no surface effects would be present. The cylinder was anchored by two strings at the nodes.

the oscillation of the pendulum. The output of the audio oscillator was used to drive an electro-magnet. The output of the audio oscillator was amplified in a power amplifier and this was used to drive the electro-magnet. The output of the crystal pickup was put into a cathode ray oscilloscope where the signal was looked for resonance. Unfortunately, due to radiation, frequencies above 1500 cycles per second could not be detected.

Two different types of frequency measurement were employed to compare the frequency of vibration, both of which gave very accurate and like results. The first method was to take the output of the pickup and put it on the vertical plates of the CRO. When a resonant signal was obtained, a picture of the frequency was taken with no horizontal sweep. The camera used was a very high speed model with no shutter. The camera had a built-in timing light of 1000 cycles per second which showed on the film. The frequency was then computed from the developed film by counting the cycles.

The other method of determining the frequency was to take the amplified output of the crystal pickup and put it into an electronic decade counter. To compute the air frequencies, the cylinder was suspended by springs located at the nodal points. The electro-magnet was placed as close as possible to the node from windings. To prevent heating of the cylinder, a rubber band was used as a standard. The frequencies were recorded as described above.

For the water tests, the cylinder was immersed seven diameters in the towing tank at the U. S. Hydrographic Laboratory. This depth assured that no surface effects would be present. The cylinder was supported by two strings at the nodes.

The same procedure was followed using cylinders of 40, 38, 30, 28, and 22 inches, and for a 42-inch bar with six-inch conical ends.

#### Analytical Procedure

The ratio of the added water mass to displaced water mass,  $M/M_0$ , was computed directly from the observed frequencies as explained in Details of Procedure.

A correction factor  $K$  was then calculated, where  $K$  is the ratio of measured  $M/M_0$  to that computed by Dr. H. M. Schauer. (2)

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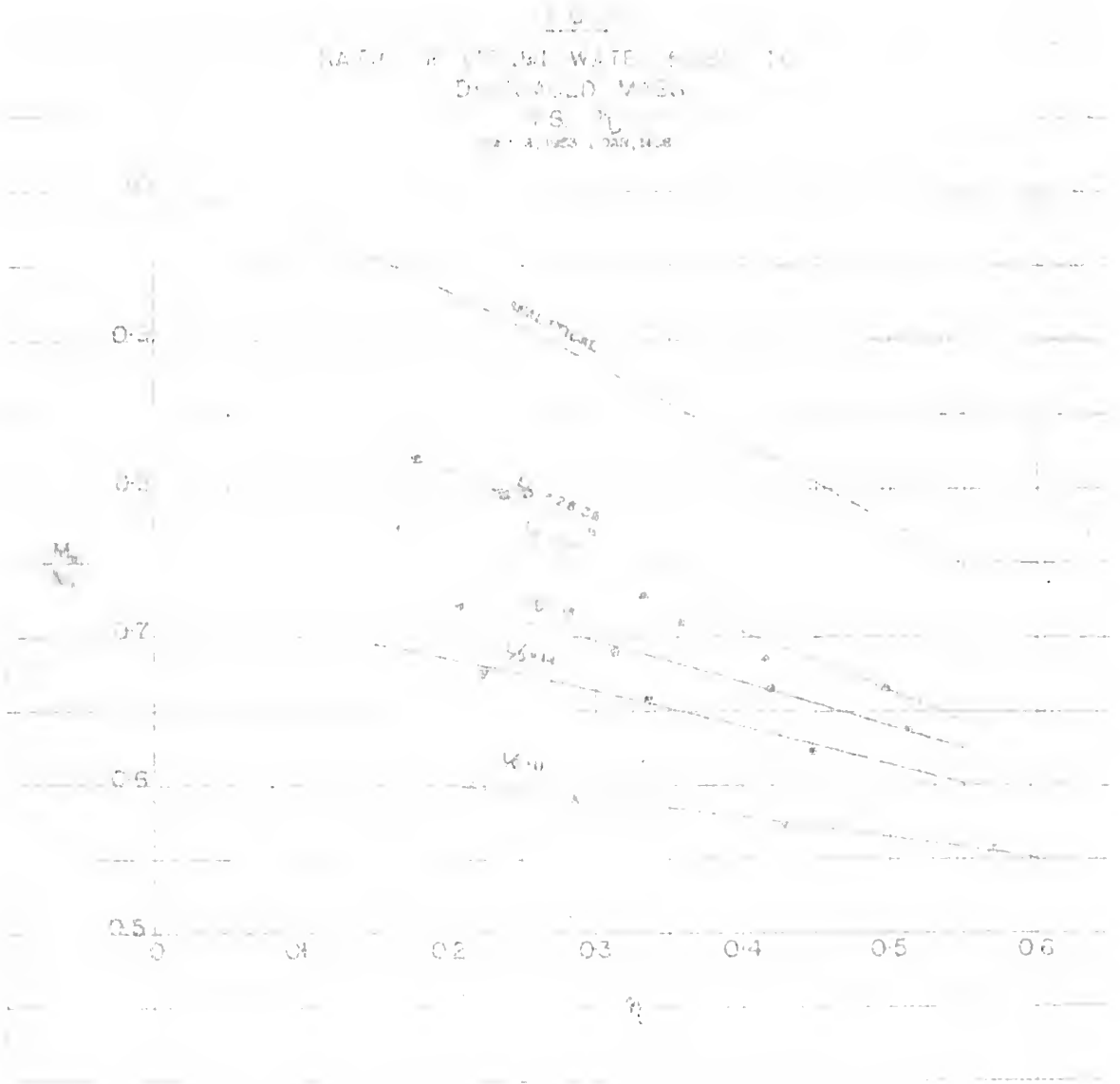
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(c) "transfer" means the transfer of the right to use the information to a third party.

### III. RESULTS

The results are plotted as shown in Figs. II through V.



-8-

III. RESULTS

The results are plotted as shown in Figs. II through V.

FIG. II

RATIO OF VIRTUAL WATER MASS TO  
DISPLACED MASS

V.S.  $\eta$

MAY 8, 1963, DAR, MCS

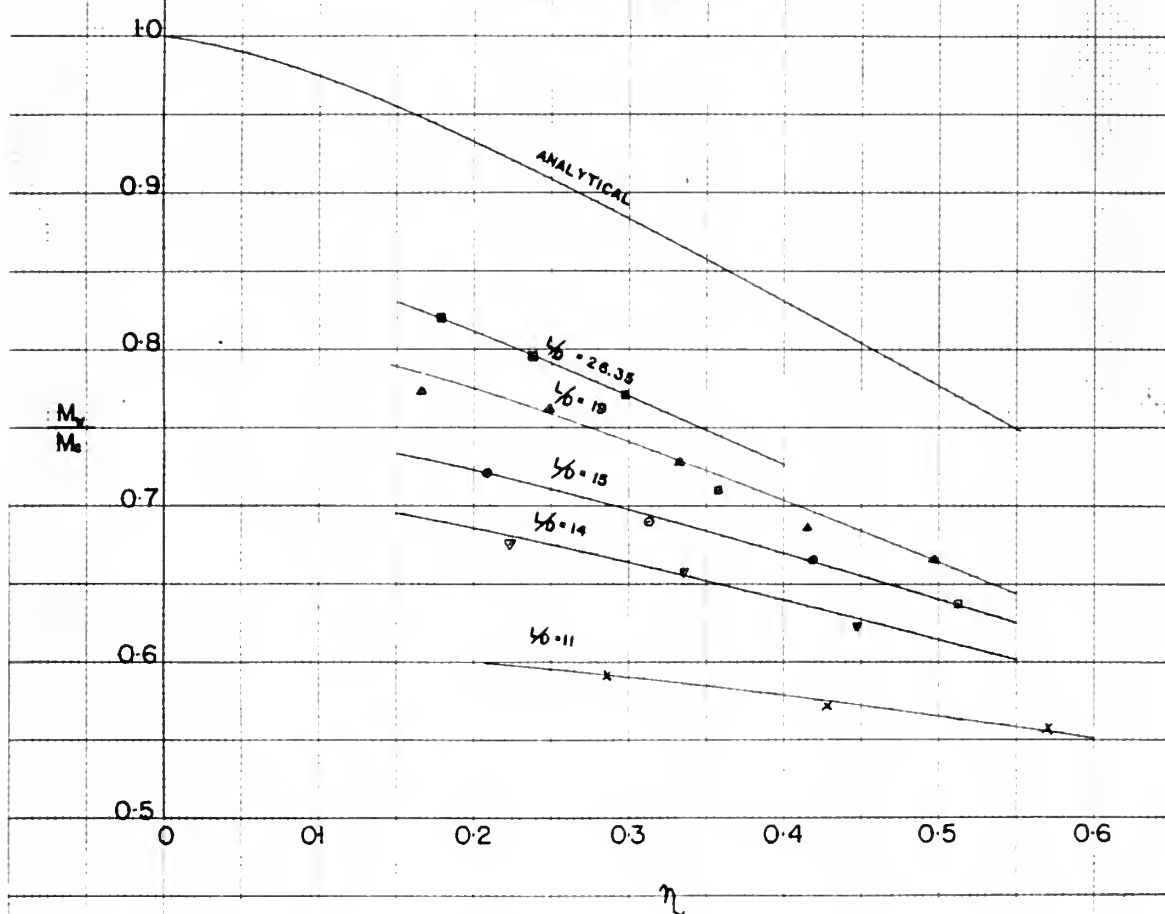






FIG. III  
MASS RATIO VS.  $L/D$  FOR  
CONSTANT  $\eta$   
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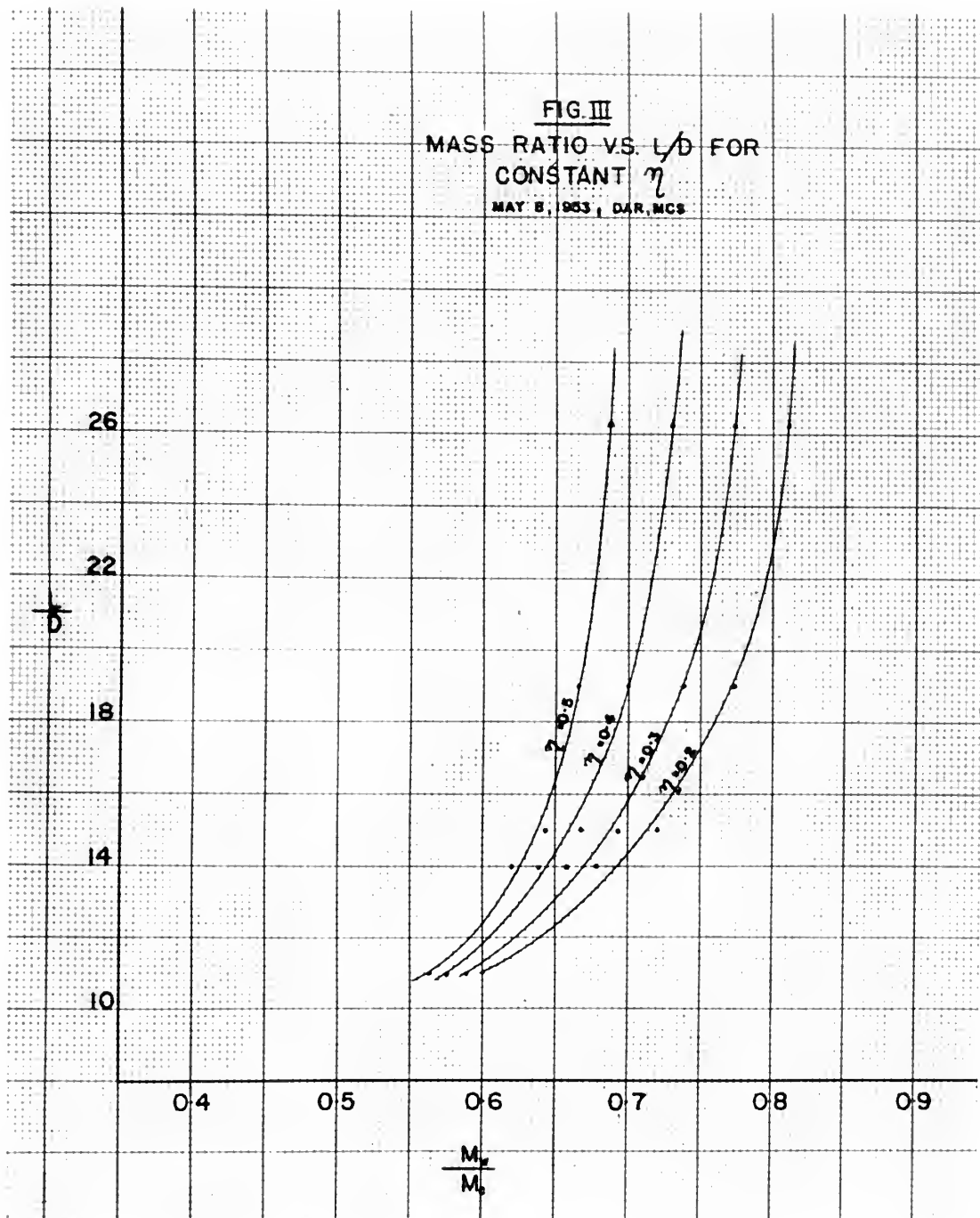




FIG. IV

K V.S.  $\eta$  FOR CONSTANT  $\frac{L}{D}$   
MAY 8, 1953, DAR, MCB

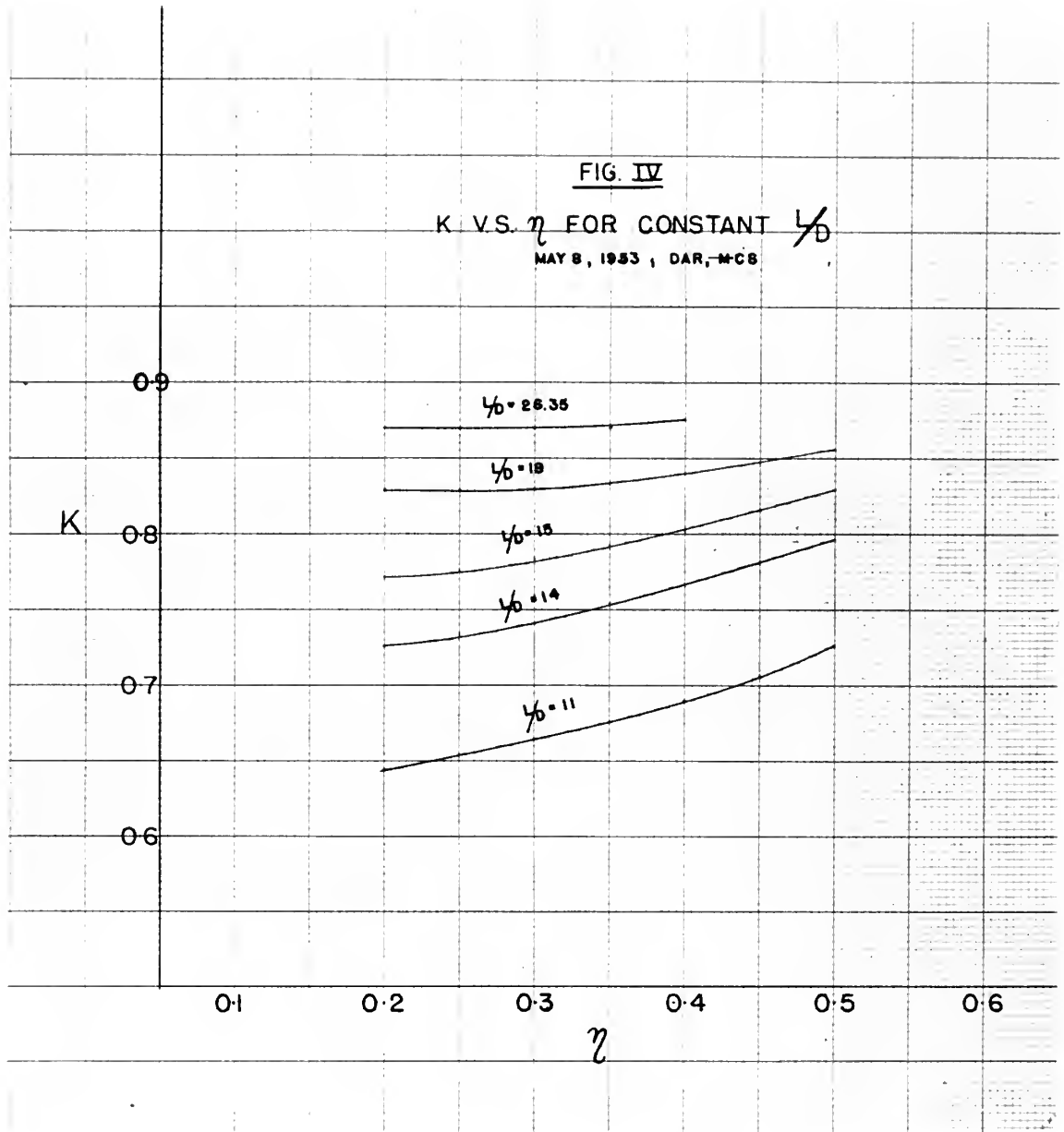
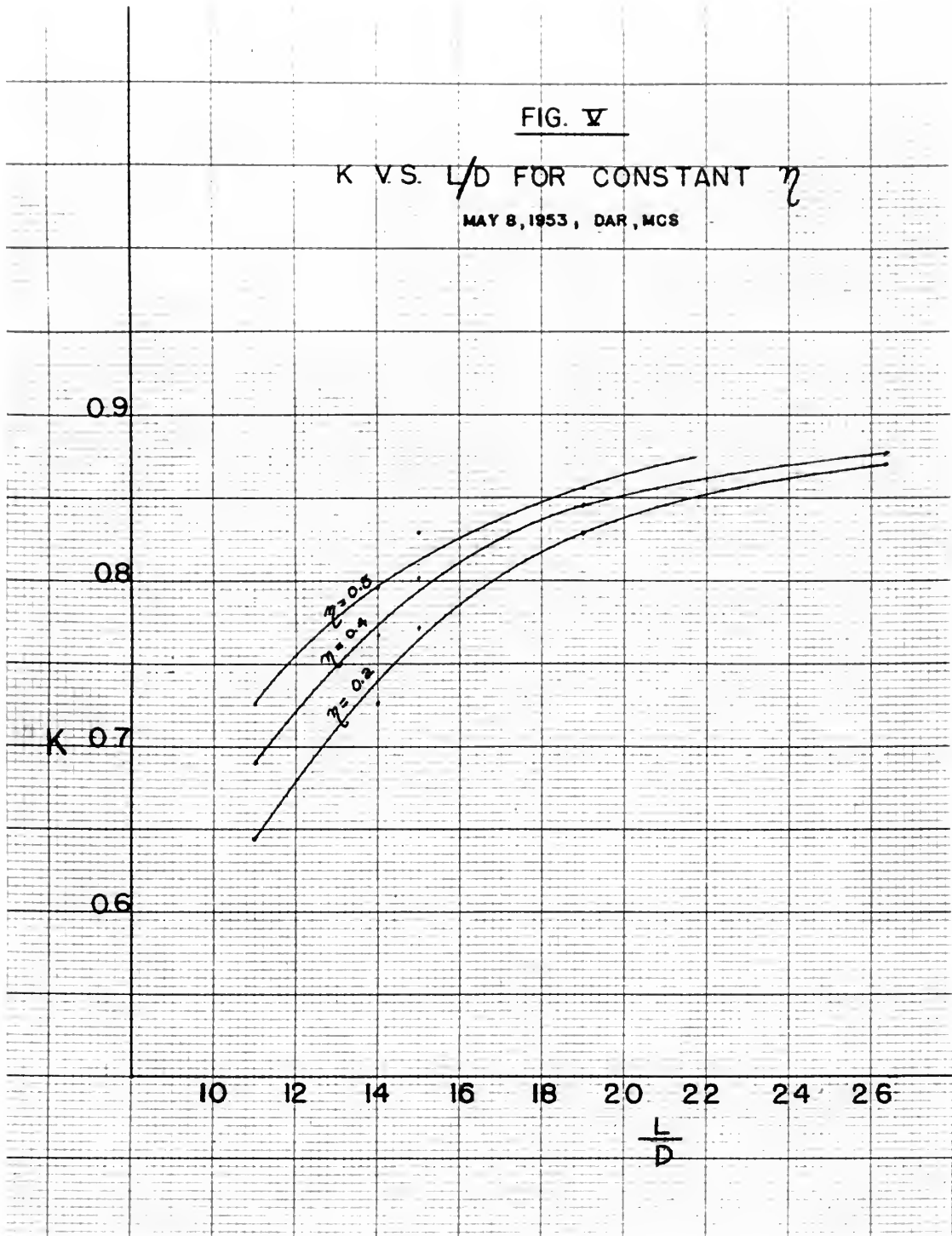




FIG. V

K V.S.  $L/D$  FOR CONSTANT  $\eta$

MAY 8, 1953, DAR, MCS





#### IV. DISCUSSION OF RESULTS

The results shown plotted in figures II through V indicate that the ends have a large influence on the value of the ratio of added water mass to the displaced water mass, i.e.  $M_w/M_c$ . As the length to diameter ratio is decreased, the value of K, (the ratio of measured  $M_w/M_c$  to the  $M_w/M_c$  calculated by Dr. Schauer), decreases rapidly for a given value of  $\eta$ . However, K increases for increasing  $\eta$  at constant L/D.

K is based on an analytical curve which assumes a potential flow, and therefore irrotational, with no flow about the ends. Thus K may be considered a measure of the end flow and of rotationality. For a cylinder with flat ends a large portion of K can probably be attributed to flow of the fluid about the ends.

Just how much of the difference from analytical values can be attributed to end flow is difficult to say from these experiments. An inspection of the test results of the cylinder with conical ends indicates that end flow is not the only contributing factor since the area at the ends of the cones is substantially zero. However, due to the abrupt change in shape of the body where the cones are joined to the cylinder, a certain amount of spilling of fluid toward the ends will occur.

Due to the large effects of L/D on the value of  $M_w/M_c$  it appears that a direct application of Dr. Schauer's equation is not feasible unless the body is very long relative to its breadth.

Professor F. M. Lewis<sup>(1)</sup> has shown that the added water mass per foot of length for an ellipsoid is





$$M_w = CJ\pi B^2 \gamma_w \quad (2)$$

where

$C$  = Section inertia coefficient

$B$  = Half beam or radius of ellipsoid at section

$\gamma_w$  = Specific weight of fluid

$J$  =  $\frac{\text{Actual K. E. surrounding fluid}}{\text{K. E. of fluid if flow is two dimensional}}$

Now for a cylinder  $C = 1$  and we can write

$$M_w = J\pi R^2 \gamma_w L \quad (3)$$

and

$$\frac{M_w}{M_o} = \frac{J\pi R^2 \gamma_w L}{\pi R^2 \gamma_w L} \quad (4)$$

so that

$$J = \frac{M_w}{M_o} \quad (5)$$

It was found that if Prof. Lewis' J factor is multiplied by our K, a fairly decent approximation to the actual frequency of a submerged body can be computed. A computation was made of the submerged frequency of a 42-inch body having 6-inch cones at each end of a cylindrical middle body 30 inches long.

Using Prof. Lewis' J factor multiplied by K for an L/D ratio of 21 at  $\eta = .15$ , the computed two noded frequency was 68.9 c.p.s. and the experimentally measured value was 70 c.p.s. For the three noded frequency the computed value was 187 c.p.s. and the measured value was 188 c.p.s.

Although this method gives very good results in the case tested, further investigation is required to determine its limits of application. Similar results would have been found using Dr. Schauer's  $M_w/M_o$  in a like manner.

(2)

where  $\gamma$  = Section inertia coefficient  
 $R$  = Half beam or radius of ellipticity of section  
 $\bar{W}$  = Specific weight of fluid  
 $\bar{W}_f$  = Specific weight of fluid  
Actual E. E. submerged fluid  
 E. E. of fluid is two dimensional

Now for a cylinder  $b = 1$  and we can write

(3)

$$M' = \frac{M}{\gamma}$$

and

(4)

$$\frac{M'}{\gamma} = \frac{M}{\gamma} - \frac{M}{\gamma}$$

(5)

$$\frac{M}{\gamma} = 1$$

so that

It was found that if  $\gamma$  factor is multiplied by our  $E$ , a fairly decent approximation to the actual frequency of a submerged body can be computed. A computation was made of the submerged frequency of a 22-inch body having 22-inch cones at each end of a cylindrical middle body 30 inches long.

Using  $\gamma$  factor multiplied by  $E$  for an elliptical cross section of 22 inches, the computed two noted frequency was 65.7 c.p.s. and the experimentally measured value was 70 c.p.s. For the three noted frequency, the computed value was 187 c.p.s. and the measured value was 198 c.p.s.

Although this method gives very good results in the case tested, further investigation is required to determine the limits of application. Further results would have been obtained using Mr. Connor's  $M'_{\gamma}$  as a line number.

#### V. RECOMMENDATIONS

In view of the large discrepancy between values of theoretical mass found by analytical means and those measured, it is recommended that an investigation similar to this be made of families of bodies of revolutions having ends whose section decreases to zero.

The measurement of the virtual mass of bodies which do not have complete radial symmetry is much more difficult since pains must be taken to insure that the vibrations are limited to the plane desired. However, it is recommended that where data exists on the vibration of submerged bodies such as submarines, an attempt be made to apply the correction,  $K$ , to compute their frequencies.

# APPENDIX I

In view of the large discrepancy between the results of the two methods

used for analytical results and direct measurement, it is recommended that an

investigation should be made of families of bodies of revolution

having ends whose section distances to axes

The measurement of the virtual mass of bodies which do not have complete

radial symmetry is much more difficult since factors must be taken to insure

that the vibrations are limited to the plane desired. However, it is

recommended that where data exists on the vibration of submerged bodies such

as submarines, an attempt be made to apply the correction,  $\frac{1}{2}$ , to compute

their frequencies.

VI. APPENDIX

VI. APPENDIX

## A. DETAILS OF PROCEDURE

### Selection of Tubing

Since the object of this investigation is to determine the virtual added mass of water, it is desirable that the cylinder be made of a material having a low specific gravity. Thus the added water mass will be a large fraction of the total virtual mass of the body. In order that the frequencies of the cylinder will not be excessively high, the material should have a relatively low modulus of elasticity.

As a result of these considerations, it was decided that the cylinder should be constructed of Lucite. The specific gravity of Lucite is 1.18 and has a modulus of elasticity in the order of  $5 \times 10^5$  pounds per square inch.

### Effect of Added Masses

The weight of the soft iron wire was .373 ounces and covered three-fourths of an inch of the cylinder. The weight of the crystal pick-up and clamp used to hold it securely in place was .303 ounces. The weight of the plastic cylinder was .497 ounces per inch. The effect of these added masses on the frequencies was assumed negligible.

### Comparison of Frequency Measurement

The high speed movie camera method of frequency measurement proved very satisfactory, but also required a great deal of time. The camera had a neon bulb timing light built in which was energized by a 1000 cycle audio oscillator. The output of the oscillator was amplified through a CRO which provided a means of adjusting the light to the desired brightness. The only errors

## APPENDIX II

### Estimation of Elasticity

Since the object of this investigation is to determine the virtual elastic mass of water, it is desirable that the cylinder be made of a material having a low specific gravity. Thus the added water mass will be a large fraction of the total virtual mass of the body. In order that the frequency of the cylinder will not be excessively high, the material should have a relatively low modulus of elasticity.

As a result of these considerations, it was decided that the cylinder should be constructed of lucite. The specific gravity of lucite is 1.18 and has a modulus of elasticity in the order of  $2 \times 10^{10}$  pounds per square inch.

### Effect of Added Masses

The weight of the soft line was .375 ounces and covered three-fourths of an inch of the cylinder. The weight of the crystal pick-up and clamp used to hold it securely in place was .307 ounces. The weight of the plastic cylinder was .197 ounces per inch. The effect of these added masses on the frequency was assumed negligible.

### Comparison of Frequency Measurements

The first series of measurements of frequency measurements proved very satisfactory, but also required a great deal of time. The camera had a bulb that required light bulb in which was energized by a 1000 cycle audio oscillator. The output of the oscillator was amplified through a GPO which provided a means of adjusting the light to the desired brightness. The only error



that could be made in this method were (a) errors in counting the film and (b) frequency drift in the oscillator. Two separate runs were made on each length tested.

In order to speed up the experiment and reduce the amount of labor involved, it was decided to try frequency measurement with an electronic decade counter. If there is any frequency drift of the audio oscillator, the results of the decade counter will be more accurate than the movie camera since the decade counter records every second noting any change which may occur.

#### Boundary Effects in Water

To check for wall effects, the cylinder was submerged in the stability tank and the test rerun. The results obtained were the same as those observed in the towing tank test. A further check was made by varying the distance of the cylinder from the walls in the stability tank. Again no difference was noted. No attempt was made to vibrate the cylinder within four diameters of the wall.

#### Comparison with Different Cylinder

The necessity of re-checking the 30-inch cylinder arose after it had been cut down to 22 inches. A new 30-inch plastic tube was re-wound with approximately the same amount of wire as before. Both the air and water tests agreed identically with the previous 30-inch test.

#### Damping Effects of Rubber Band Stand-off

To ascertain whether the rubber band had any appreciable damping effect, the force necessary to stretch the rubber band one inch and the force required to deflect the tube one inch were calculated. To stretch the rubber band

... ..  
... ..  
... ..

In order to speed up the experiment and avoid the amount of labor involved, it was decided to try frequency measurement with an electronic decade counter. It was in my frequent mind of the radio oscillator the results of the decade counter will be more accurate than the more exact also the decade counter records every second noting any change which may occur.

Frequency Effects in Water

In check for wall effects, the cylinder was submerged in the stability tank and the test run. The results obtained were the same as those observed in the towing tank test. A further check was made by varying the distance of the cylinder from the walls in the stability tank. Again no difference was noted. A distance was made to vibrate the cylinder within four thousandths of the wall.

Comparison with Literature

The necessity of re-entranting the 30-inch cylinder when it had been out from 15 inches. A new 30-inch plastic pipe was removed with approximately the same amount of stress as before. Both the air and water tests agreed satisfactorily with the previous 30-inch test.

Resonance Effects of Water in the Tank

To determine whether the water in the tank had any appreciable damping effect, the lower cylinder was struck the water tank and the lower cylinder was struck the water tank. The results were similar to the upper cylinder.

one inch required a force of .61 pounds, and to deflect the cylinder one inch, a force of 454 pounds was necessary. The damping effect of the rubber band may be neglected.

#### Amplitude Measurement

The equipment used was not capable of measuring amplitudes. This is partly because the natural resonant period of the pick-up, 980 cycles per second, was within the range of the frequencies measured. The primary difficulty was that it was not possible to maintain a constant exciting force on the cylinder. There also was no means of accurately measuring the exciting force. A constant exciting force was not possible because a higher force was required to excite the higher modes, but this same force would cause the cylinder to be drawn hard against the faces of the electro-magnet poles at the lower modes. It was necessary therefore to start with relatively low driving forces at low modes and increase the force to excite the higher frequencies.

#### Computation of Virtual Mass

The virtual mass was computed assuming negligible damping so that

$$\frac{M_V}{M_T} = \left( \frac{f_1}{f_2} \right)^2 \quad (6)$$

where

$M_V$  = Virtual Mass

$M_T$  = Mass of Tube

$f_1$  = Frequency in Air

$f_2$  = Frequency in Water

... of the ...  
 ... of the ...  
 ... of the ...

## Assumptions

The equivalent circuit was not capable of representing a circuit. This is partly because the natural frequency of the circuit, 20 cycles per second, was within the range of the frequency response of the circuit. It is difficult to see that it was not possible to maintain a constant exciting force on the system. There also was no means of accurately measuring the exciting force. A constant exciting force was not possible because a higher force was required to excite the higher modes but this same force would cause the circuit to be driven into resonance. The force of the electro-magnetic poles at the lower modes. It was necessary therefore to start with relatively low driving forces at low order and increase the force to excite the higher frequencies.

## Calculation of Virtual Mass

The virtual mass was calculated assuming negligible damping so that

(d)

$$\left( \frac{1}{Z} \right) = \frac{1}{Z_1} + \frac{1}{Z_2}$$

where

- $Z_1$  = Virtual Mass
- $Z_2$  = Mass of Water
- $Z_3$  = Frequency in Hz
- $Z_4$  = Frequency in Hz

Let

$M_w$  = Added Water Mass

$M_c$  = Displaced Water Mass

then

$$M_v = M_w + M_T \quad (7)$$

$$M_w = M_T \left[ \left( \frac{f_1}{f_2} \right)^2 - 1 \right] \quad (8)$$

and

$$\frac{M_w}{M_c} = \frac{M_T}{M_c} \left[ \left( \frac{f_1}{f_2} \right)^2 - 1 \right] \quad (9)$$

Thus the ratio  $\left( \frac{M_w}{M_c} \right)$  may be computed from the measured values of the frequencies.

#### Computation of K

Dr. H. M. Schauer<sup>(2)</sup> has derived an analytical expression for the mass ratio,  $\frac{M_w}{M_c}$ , as follows:

$$\frac{M_w}{M_c} = \frac{1}{1 + \eta \frac{iH_0(i\eta)}{-H_1(i\eta)}} \quad (10)$$

where  $H_0$  and  $H_1$  are Hankel functions and other symbols as previously defined.

The variation of the measured mass ratio from the above will be a function of  $\eta$  and the  $\frac{L}{D}$  ratio. This ratio can be expressed as a ratio, K

$$K = \frac{\text{measured mass ratio}}{\text{analytical mass ratio}}$$

$$= \frac{(M_w/M_c)_M}{(M_w/M_c)_A} \quad (11)$$

the total value of  $N_V$  is  $N_V = N_V^0 + N_V^1$  and the value of  $N_V^0$  is  $N_V^0 = N_V^1 \left[ \left( \frac{1}{2} \right)^2 - 1 \right]$

then

$$N_V = N_V^0 + N_V^1$$

$$N_V = N_V^1 \left[ \left( \frac{1}{2} \right)^2 - 1 \right]$$

and

$$\frac{N_V}{N_0} = \frac{N_V^1}{N_0} \left[ \left( \frac{1}{2} \right)^2 - 1 \right]$$

Thus the ratio  $\left( \frac{N_V}{N_0} \right)$  may be computed from the measured values of the

frequencies.

### Computation of $K$

Dr. W. N. Gubner (2) has derived an analytical expression for the mass

ratio,  $\frac{M_V}{M_0}$ , as follows:

$$\frac{M_V}{M_0} = \frac{1}{1 - \frac{1}{2} \left( \frac{H_1}{H_0} \right)^2} \left[ \frac{1}{1 - \frac{1}{2} \left( \frac{H_1}{H_0} \right)^2} \right]$$

where  $H_0$  and  $H_1$  are Hankel functions and other symbols as previously defined.

The variation of the measured mass ratio from the above will be a func-

tion of  $\frac{H_1}{H_0}$  and the  $\frac{1}{2}$  ratio. This ratio can be expressed as a ratio,  $K$

$K = \frac{\text{measured mass ratio}}{\text{analytical mass ratio}}$

$$K = \frac{\left( \frac{M_V}{M_0} \right)}{\left( \frac{M_V}{M_0} \right)_{\text{analytical}}}$$

B. SUMMARY OF DATA AND CALCULATIONS

1. Definition of Symbols.

$f_1$  = Observed air frequency, cycles per second

$f_2$  = Observed water frequency, cycles per second

$M_w$  = Added mass of water

$M_o$  = Mass of displaced water

$L$  = Length of cylinder, inches

$D$  = Diameter of cylinder, inches

$\eta = \frac{D}{2L} (m + 1)$

$m$  = Mode number

# THEORY OF THE EARTH

CHAPTER I

1. The Earth is a sphere, and its surface is divided into two parts, the land and the water.

2. The land is divided into continents and islands.

3. The water is divided into oceans and seas.

4. The land is divided into countries and provinces.

5. The water is divided into rivers and lakes.

$$1 + 2 = 3$$

6. The Earth is a sphere, and its surface is divided into two parts, the land and the water.



2. Calculated values of  $\frac{M_v}{M_c}$ .

MODE	$f_1$	$f_2$	$\frac{M_v}{M_c}$	$\eta$
1	61.7			
2	170.5	86.2	0.82	0.179
3	332.0	169.1	0.795	0.238
4	539.5	279.0	0.770	0.298
5	787.5	419.0	0.709	0.358

TABLE I

Test Results 52.7-inch Cylinder  $L/D = 26.35$

---

MODE	$f_1$	$f_2$	$\frac{M_v}{M_c}$	$\eta$
1	104.5	54	.792	.157
2	290.5	151	.768	.236
3	562	295	.742	.314
4	893	476	.711	.392
5	1277	692	.682	.471

TABLE II

Test Results of 40-inch cylinder  $L/D = 20$



MODE	$f_1$	$f_2$	$\frac{M}{M_0}$	$\eta$
1	114	59	0.773	0.166
2	313	163	0.761	0.249
3	602	319	0.727	0.332
4	943	509	0.685	0.415
5	1351	739.5	0.665	0.497

TABLE IIITest Results 38-inch Cylinder  $L/D = 19$ 


---

MODE	$f_1$	$f_2$	$\frac{M}{M_0}$	$\eta$
1	183	97.5	0.720	0.209
2	496.5	268.5	0.689	0.314
3	936	512	0.665	0.419
4	1454	808	0.637	0.523

Table IV

Test Results 30-inch Cylinder  $L/D = 15$

MODE	$I_1$	$I_2$	$\frac{M}{M_0}$	$I$
1	174	50	0.772	0.166
2	313	163	0.767	0.248
3	602	312	0.763	0.338
4	943	502	0.682	0.412
5	1351	732.5	0.602	0.487

TABLE III

Test Results on 30-Kilon Cylinder,  $I_0 = 19$

MODE	$I_1$	$I_2$	$\frac{M}{M_0}$	$I$
1	183	47.5	0.750	0.109
2	402.5	202.5	0.689	0.214
3	636	312	0.662	0.412
4	1424	808	0.623	0.523

Table IV

Test Results on 30-Kilon Cylinder,  $I_0 = 19$

MODE	$f_1$	$f_2$	$\frac{M}{M_0}$	$\eta$
1	205	112	0.674	0.224
2	553	305	0.657	0.336
3	1036	581.5	0.622	0.448

TABLE V

Test Results 28-inch Cylinder  $L/D = 14$

---

MODE	$f_1$	$f_2$	$\frac{M}{M_0}$	$\eta$
1	299	172	0.590	0.286
2	800	465	0.570	0.428
3	1457	855	0.556	0.571

TABLE VI

Test Results 22-inch Cylinder  $L/D = 11$

Run	$\frac{M_w}{M_n}$	$\bar{P}_n$	$\bar{P}_w$	MOE
1	0.874	113	202	1
2	0.821	202	327	2
3	0.823	201.2	1026	3

TABLE V

Test Results 35-inch Cylinder  $\sqrt{D} = 1.8$

Run	$\frac{M_w}{M_n}$	$\bar{P}_n$	$\bar{P}_w$	MOE
1	0.820	172	299	1
2	0.810	242	300	2
3	0.811	222	1421	3

TABLE VI

Test Results 35-inch Cylinder  $\sqrt{D} = 1.8$

$\eta$	$1H_o(1\eta)$	$-H_1(1\eta)$	$\frac{1H_o(1\eta)}{-H_1(1\eta)}$	$\eta \frac{1H_o(1\eta)}{-H_1(1\eta)}$	$1+\eta \frac{1H_o(1\eta)}{-H_1(1\eta)}$	$\frac{M_w}{M_o}$
.120	1.431	5.20	.275	.0330	1.0330	.968
.131	1.376	4.75	.290	.0380	1.0380	.963
.175	1.198	3.50	.342	.0599	1.0599	.943
.196	1.128	3.11	.363	.0711	1.0711	.934
.261	.9956	2.275	.438	.114	1.114	.898
.262	.9954	2.265	.439	.115	1.115	.897
.327	.8238	1.763	.437	.153	1.153	.867
.349	.7865	1.635	.481	.168	1.168	.856
.392	.7209	1.425	.506	.198	1.198	.835
.437	.6609	1.249	.529	.231	1.231	.812
.524	.5639	.9928	.568	.298	1.298	.770

TABLE VIII

Analytical Calculation of Virtual Mass

By Dr. H. M. Schauer<sup>(2)</sup>

$$\frac{M_w}{M_o} = \frac{1}{1 + \eta \frac{1H_o(1\eta)}{-H_1(1\eta)}}$$

$M_w$  = Added Water Mass

$M_o$  = Mass of Displaced Water

$M_T$  = Mass of Cylinder

$$\eta = (n+1) \frac{a}{L}$$

$n$  = Mode Number

$a$  = Radius

$L$  = Length

Run	$\frac{M_w}{M_n}$	$\bar{P}_n$	$\bar{P}_w$	Notes
1	0.874	172	202	
2	0.877	202	222	
3	0.882	201.2	1020	

TABLE V

Test Results 30-inch Cylinder  $\bar{P}_n = 11$

Run	$\frac{M_w}{M_n}$	$\bar{P}_n$	$\bar{P}_w$	Notes
1	0.880	172	222	
2	0.870	202	202	
3	0.871	202	1020	

TABLE VI

Test Results 30-inch Cylinder  $\bar{P}_n = 11$



$\eta$	$1H_o(1\eta)$	$-H_1(1\eta)$	$\frac{1H_o(1\eta)}{-H_1(1\eta)}$	$\eta \frac{1H_o(1\eta)}{-H_1(1\eta)}$	$1+\eta \frac{1H_o(1\eta)}{-H_1(1\eta)}$	$\frac{M_W}{M_o}$
.120	1.431	5.20	.275	.0330	1.0330	.968
.131	1.376	4.75	.290	.0380	1.0380	.963
.175	1.198	3.50	.342	.0599	1.0599	.943
.196	1.128	3.11	.363	.0711	1.0711	.934
.261	.9956	2.275	.438	.114	1.114	.898
.262	.9954	2.265	.439	.115	1.115	.897
.327	.8238	1.763	.437	.153	1.153	.867
.349	.7865	1.635	.481	.168	1.168	.856
.392	.7209	1.425	.506	.198	1.198	.835
.437	.6609	1.249	.529	.231	1.231	.812
.524	.5639	.9928	.568	.298	1.298	.770

TABLE VIII

Analytical Calculation of Virtual Mass

By Dr. H. M. Schauer<sup>(2)</sup>

$$\frac{M_W}{M_o} = \frac{1}{1 + \eta \frac{1H_o(1\eta)}{-H_1(1\eta)}}$$

$M_W$  = Added Water Mass

$M_o$  = Mass of Displaced Water

$M_T$  = Mass of Cylinder

$$\eta = (n+1) \frac{a}{L}$$

$n$  = Mode Number

$a$  = Radius

$L$  = Length

$\frac{M}{M_0}$	$\frac{M}{M_0} \left( \frac{1}{1 - \frac{1}{2} \frac{v^2}{c^2}} \right)$	$\frac{M}{M_0} \left( \frac{1}{1 - \frac{1}{2} \frac{v^2}{c^2}} \right)^2$	$\frac{M}{M_0} \left( \frac{1}{1 - \frac{1}{2} \frac{v^2}{c^2}} \right)^3$	$\frac{M}{M_0} \left( \frac{1}{1 - \frac{1}{2} \frac{v^2}{c^2}} \right)^4$	$\frac{M}{M_0} \left( \frac{1}{1 - \frac{1}{2} \frac{v^2}{c^2}} \right)^5$	$\frac{M}{M_0} \left( \frac{1}{1 - \frac{1}{2} \frac{v^2}{c^2}} \right)^6$
0.99	1.0000	0.9900	0.9703	0.9414	0.9034	0.8563
0.98	1.0080	0.9880	0.9680	0.9380	0.9000	0.8530
0.97	1.0160	0.9960	0.9760	0.9460	0.9080	0.8610
0.96	1.0240	1.0040	0.9840	0.9540	0.9160	0.8690
0.95	1.0320	1.0120	0.9920	0.9620	0.9240	0.8770
0.94	1.0400	1.0200	1.0000	0.9700	0.9320	0.8850
0.93	1.0480	1.0280	1.0080	0.9780	0.9400	0.8930
0.92	1.0560	1.0360	1.0160	0.9860	0.9480	0.9010
0.91	1.0640	1.0440	1.0240	0.9940	0.9560	0.9090
0.90	1.0720	1.0520	1.0320	1.0020	0.9640	0.9170
0.89	1.0800	1.0600	1.0400	1.0100	0.9720	0.9250
0.88	1.0880	1.0680	1.0480	1.0180	0.9800	0.9330
0.87	1.0960	1.0760	1.0560	1.0260	0.9880	0.9410
0.86	1.1040	1.0840	1.0640	1.0340	0.9960	0.9490
0.85	1.1120	1.0920	1.0720	1.0420	1.0040	0.9570
0.84	1.1200	1.1000	1.0800	1.0500	1.0120	0.9650
0.83	1.1280	1.1080	1.0880	1.0580	1.0200	0.9730
0.82	1.1360	1.1160	1.0960	1.0660	1.0280	0.9810
0.81	1.1440	1.1240	1.1040	1.0740	1.0360	0.9890
0.80	1.1520	1.1320	1.1120	1.0820	1.0440	0.9970

# APPENDIX

## Calculation of Virtual Mass

By Dr. J. H. Schaefer

$$\frac{M}{M_0} = \frac{1}{1 - \frac{1}{2} \frac{v^2}{c^2}}$$

$M_0$  = Mass of body

$M$  = Mass of displaced water

$M_1$  = Mass of cylinder

$$\frac{M}{M_0} = \frac{1}{1 - \frac{1}{2} \frac{v^2}{c^2}}$$

$v$  = velocity

$c$  = velocity of light

$\frac{M}{M_0}$  = virtual mass

3. Calculated values of K.

$\eta$	$(M_w/M_o)_M$	$(M_w/M_o)_A$	K
.2	.811	.933	.870
.25	.792	.910	.870
.30	.772	.886	.870
.35	.750	.860	.870
.40	.730	.833	.876

TABLE IX (a)

Values of K  $L/D = 26.35$

---

$\eta$	$(M_w/M_o)_M$	$(M_w/M_o)_A$	K
.2	.772	.933	.829
.25	.754	.910	.829
.30	.737	.886	.830
.35	.718	.810	.834
.40	.700	.833	.840
.45	.683	.805	.847
.50	.665	.776	.856

TABLE IX (b)

Values of K  $L/D = 19$

1	2	3	4
078.	889.	178.	8.
078.	010.	289.	28.
078.	818.	377.	08.
078.	010.	077.	28.
078.	808.	087.	10.

(2) XT 818-7

Values of X 10 = 10.99

1	2	3	4
058.	889.	177.	8.
058.	010.	277.	28.
058.	889.	787.	08.
058.	009.	810.	28.
048.	878.	007.	14.
787.	808.	888.	24.
058.	177.	888.	08.

(3) XT 818-7

Values of X 10 = 10.99

$\eta$	$(M_W/M_S)_H$	$(M_W/M_S)_A$	K
.2	.72	.933	.772
.25	.707	.910	.776
.30	.694	.886	.784
.35	.682	.860	.793
.40	.668	.833	.802
.45	.656	.805	.816
.50	.643	.776	.829

TABLE IX (c)

Values of K  $L/D = 15$

---

$\eta$	$(M_W/M_S)_H$	$(M_W/M_S)_A$	K
.2	.677	.933	.726
.25	.667	.910	.734
.30	.657	.886	.741
.35	.648	.860	.754
.40	.639	.833	.767
.45	.629	.805	.781
.50	.619	.776	.796

TABLE IX (d)

Values of K  $L/D = 14$

$\lambda$	$\lambda \sqrt{\frac{M}{\mu}}$	$\lambda \sqrt{\frac{M}{\mu}}$	$\lambda$
270.	833.	85.	2.
275.	840.	86.	2.5.
280.	848.	87.	3.
285.	856.	88.	3.5.
290.	863.	89.	4.
295.	871.	90.	4.5.
300.	878.	91.	5.

TABLE IX (c)

Values of  $F$  for  $\lambda = 12$

$\lambda$	$\lambda \sqrt{\frac{M}{\mu}}$	$\lambda \sqrt{\frac{M}{\mu}}$	$\lambda$
270.	833.	85.	2.
275.	840.	86.	2.5.
280.	848.	87.	3.
285.	856.	88.	3.5.
290.	863.	89.	4.
295.	871.	90.	4.5.
300.	878.	91.	5.

TABLE IX (d)

Values of  $F$  for  $\lambda = 12$

$\eta$	$(M_V/M_O)_M$	$(M_V/M_O)_A$	K
.2	.600	.933	.644
.25	.595	.910	.654
.30	.590	.886	.664
.35	.585	.860	.676
.40	.577	.833	.690
.45	.571	.805	.706
.50	.565	.776	.726

TABLE IX (e)

Values of K  $L/D = 11$

---

$\eta$	$(M_V/M_O)_M$	$(M_V/M_O)_A$	K
.15	.800	.954	.839
.224	.782	.922	.849
.30	.753	.886	.849
.40	.712	.833	.855
.50	.674	.776	.869

TABLE IX (f)

Values of K  $L/D = 21$

$\gamma$	$\lambda \gamma \sqrt{V}$	$\mu \gamma \sqrt{V}$	$\beta$
100.	887.	203.	0.
110.	897.	207.	25.
120.	908.	209.	50.
130.	918.	212.	75.
140.	928.	217.	100.
150.	938.	221.	125.
160.	947.	225.	150.

### (6) ALUMINUM

Values of  $\lambda \gamma \sqrt{V}$  to  $\mu \gamma \sqrt{V}$

$\gamma$	$\lambda \gamma \sqrt{V}$	$\mu \gamma \sqrt{V}$	$\beta$
100.	179.	408.	25.
110.	180.	409.	50.
120.	181.	410.	75.
130.	182.	411.	100.
140.	183.	412.	125.
150.	184.	413.	150.

### (7) ALUMINUM

Values of  $\lambda \gamma \sqrt{V}$  to  $\mu \gamma \sqrt{V}$



### C. SAMPLE CALCULATIONS

1. Theoretical frequency of free-free bar vibrating in air.

$$f = \frac{m_n^2}{2\pi} \left[ \frac{EK^2g}{\gamma} \right]^{1/2}$$

where  $m_n = 4.730$  for first mode

$E$  = Modulus of elasticity

= 580,000 psi for Lucite (approx.)

$\gamma$  = Specific gravity = 1.18

$g = 386 \text{ in/sec}^2$

$k$  = Radius of gyration of section

$k^2 = .441$

$$f = \frac{.0081}{6.28} \frac{580,000 \times .441}{11.05 \times 10^{-5}}^{1/2}$$

= 62.0 cycles per second

for 2 noded frequency of 52.7-inch cylinder.

2. Calculation of  $\frac{M_w}{M_c}$ .

$$\frac{M_w}{M_c} = \frac{M_T}{M_c} \left[ \left( \frac{f_1}{f_2} \right)^2 - 1 \right]$$

where  $M_w$  = Added water mass

$M_c$  = Displaced water mass

$M_T$  = Mass of tube

$$T = \frac{1}{2} \left( \frac{1}{\omega} + \frac{1}{\omega'} \right)$$

1. Theoretical frequency of vibration of a string fixed at both ends is given by

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

where  $f_n$  = frequency of vibration (Hz)  
 $n$  = number of antinodes  
 $L$  = length of string (m)  
 $T$  = tension in string (N)  
 $\mu$  = mass per unit length (kg/m)  
 $\mu = \frac{m}{L}$   
 $m$  = mass of string (kg)  
 $L$  = length of string (m)

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

For a string fixed at both ends, the frequency of vibration is given by

2. Calculation of  $f_n$

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$

where  $f_n$  = frequency of vibration (Hz)  
 $n$  = number of antinodes  
 $L$  = length of string (m)  
 $T$  = tension in string (N)  
 $\mu$  = mass per unit length (kg/m)

For second mode of 52.7-inch bar

$$W_T = Wt./in. \times L + Wt. \text{ clamp} + Wt. \text{ Pickup} + Wt. \text{ Exciting wire}$$

$$M_T = \frac{.497 \times 52.7 + .373 + .303}{16 \times 32.2}$$

$$= 0.0521$$

$$M_e = \frac{\pi D^2 L \gamma_w}{4g}$$

$$= \frac{3.14 \times 4 \times 52.7 \times 62.4}{4 \times 32.2 \times 1728}$$

$$= 0.186$$

$$f_1 = 170.5$$

$$f_2 = 86.2$$

$$\frac{M_w}{M_e} = .280(2.92)$$

$$= 0.820$$

### 3. Calculation of K.

By definition

$$K = \frac{M_w/M_e \text{ measured}}{M_w/M_e \text{ analytically computed}}$$

For 52.7" cylinder

when  $\eta = .2$

$$\left(\frac{M_w}{M_e}\right)_M = .811$$

$$\left(\frac{M_w}{M_e}\right)_A = .933$$

Hence  $K = \frac{.811}{.933} = 0.870$

1. Calculation of  $K$

By definition  $K = \frac{M}{W} \times 100$  where  $M$  is the mass of the sample and  $W$  is the weight of the sample.

$$K = \frac{0.001 \times 100}{0.001} = 100$$

$$K = 100 \times 100$$

$$K = 10000$$

$$K = \frac{0.001 \times 100}{0.001} = 100$$

$$K = \frac{0.001 \times 100}{0.001} = 100$$

$$K = 100 \times 100$$

$$K = 10000$$

$$K = 100 \times 100$$

$$K = 100 \times 100$$

$$K = \frac{0.001 \times 100}{0.001} = 100$$

$$K = 10000$$

### 3. Calculation of $K$

By definition

$$K = \frac{M}{W} \times 100$$

For 50% dilution

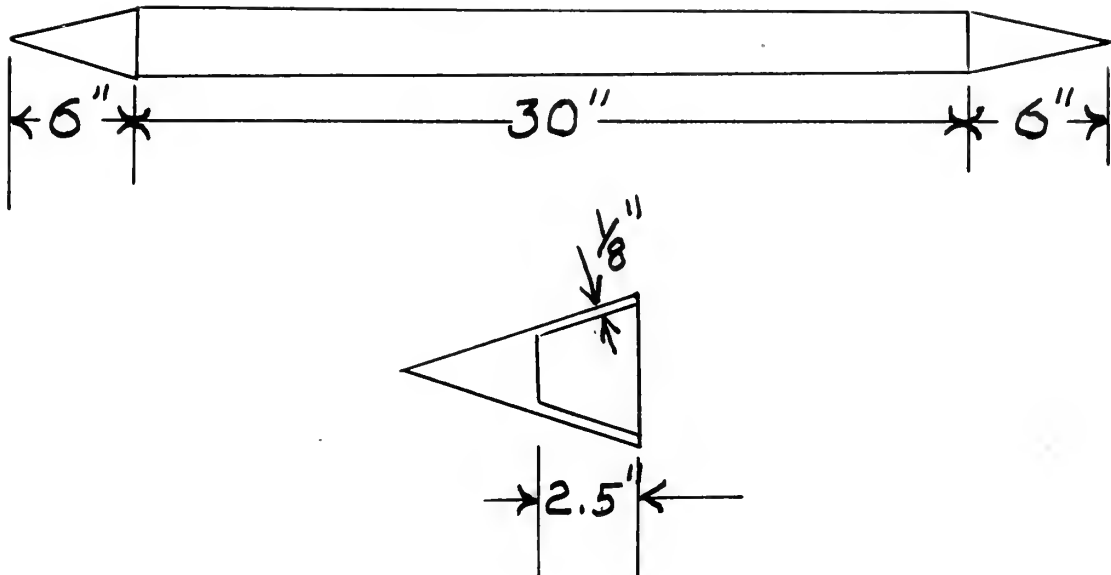
when

$$K = \frac{M}{W} \times 100$$

$$K = \frac{M}{W} \times 100$$

$$K = \frac{M}{W} \times 100$$

4.



The center of gravity of cones is approximately 3" from the base. Let us assume this body to be equivalent to a right circular cylinder 36 inches in length.

This assumption may be checked by calculating the air frequency of a 36-inch cylinder.

$$f = \frac{n^2}{2\pi} \left( \frac{EK^2}{\gamma} \right)^{1/2}$$

$$\text{where } \frac{1}{2\pi} \left( \frac{EK^2}{\gamma} \right)^{1/2} = 7,350$$

$$n = \frac{4,730}{L} \text{ for 2 noded frequency}$$

$$f = \left( \frac{4,730}{36} \right)^2 7350$$

$$f = 127 \text{ cps}$$

Observed frequency was 131 cps, therefore equivalent length is 35.5 inches.



The object is a cylinder of length 10 inches and diameter 2 inches. The cylinder is shown in perspective view. The top and bottom elliptical bases are drawn with dashed lines to indicate hidden portions. The length of the cylinder is dimensioned as 10 inches (10''). The diameter of the bases is dimensioned as 2 inches (2'').

The cylinder is shown in elevation view. The cylinder is shown as a rectangle with a triangular section removed from the top right corner. The total height of the cylinder is dimensioned as 4 inches (4''). The width of the base is dimensioned as 2 inches (2''). The slanted cut surface is dimensioned with a length of 3 inches (3'').



The water frequency now may be computed using Prof. Lewis' J, corrected by K for a 42-inch right circular cylinder

$$f = \left( \frac{4.730}{35.5} \right)^2 (7350) \left( \frac{M_T}{M_T + M_W} \right)^{1/2}$$

where  $M_T = .036$

$$M_W = JKM_c \quad (12)$$

$$= (.947) (.839) (.120)$$

$$M_W = .0952$$

$$M_W + M_T = .1312$$

$$f = 68.9 \text{ cps.}$$

The experimental results gave a water frequency of 70 cps.

For the second mode

$$J = .923$$

$$K = .849$$

$$M_W = .0940$$

$$M_W + M_T = .1300$$

$$f = 187 \text{ cps}$$

Experimental results gave a water frequency of 188 cps.

Substituting  $\frac{1}{2}$  for  $\frac{1}{2}$  in the above equation, we get

which is a quadratic equation in  $\frac{1}{2}$ .

$$\frac{1}{2} \left( \frac{1}{2} \right) = \frac{1}{2} \left( \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} \right)$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

(15)

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$(0.5) (0.5) = (0.5) (0.5) + (0.5) (0.5) + (0.5) (0.5)$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

The experimental results gave a value frequency of 0.5.

For the second note

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

Experimental results gave a value frequency of 0.5.



D. ORIGINAL DATA

1. The following data was taken using the decade counter.

<u>MODE</u>	<u>FREQUENCIES</u>			<u>AVERAGE</u>
1	114	114	114	114
2	313	313	313	313
3	604	601	602	602
4	944	942	942	943
5	1352	1350	1351	1351

TABLE X

Air Frequencies 38-inch Cylinder

---

<u>MODE</u>	<u>FREQUENCIES</u>			<u>AVERAGE</u>
1	59	59	59	59
2	163	163	163	163
3	320	319	319	319
4	510	507	509	509
5	740	739	739.5	739.5

TABLE XI

Water Frequencies 38-inch Cylinder

# TABLE X

The following data were taken from the records compiled.

TIME	TEMPERATURE	WIND	WIND DIRECTION	WIND VELOCITY
1	114	114	114	114
2	113	113	113	113
3	104	104	104	104
4	104	104	104	104
5	132	132	132	132

# TABLE XI

Air Temperature 38-Inch Cylinder

TIME	TEMPERATURE	WIND	WIND DIRECTION	WIND VELOCITY
1	114	114	114	114
2	113	113	113	113
3	104	104	104	104
4	104	104	104	104
5	132	132	132	132

# TABLE XII

Water Temperature 38-Inch Cylinder

MODE	FREQUENCIES				AVERAGE
1	182	183	183	183	183
2	496.5	495.5	496.5	496.5	496.5
3	933	936	935.5	936.5	936.0
4	1454.5	1453.5	1453	1454.5	1454.0

TABLE XII

Air Frequencies 30-inch Cylinder

---

MODE	FREQUENCIES				AVERAGE
1	98	97.5	97.5	97.5	97.5
2	268.5	269	267	268.5	268.5
3	510	513	513.5	511	512.0
4	810	809	808	807.5	808

TABLE XIII

Water Frequencies 30-inch Cylinder

TIME	TEMPERATURE	AVG	TEMPERATURE	AVG
1	185	185	185	185
2	185	185	185	185
3	185	185	185	185
4	185	185	185	185

### TABLE III

Air Properties 30-inch Cylinder

MODE	TEMPERATURE	AVG	TEMPERATURE	AVG
1	185	185	185	185
2	185	185	185	185
3	185	185	185	185
4	185	185	185	185

### TABLE III

Air Properties 30-inch Cylinder

MODE	FREQUENCIES		AVERAGE
1	205	205	205
2	553	552.5	553
3	1036	1035.5	1036

TABLE XIV

Air Frequencies 28-inch Cylinder

---

MODE	FREQUENCIES		AVERAGE
1	112	112	112
2	305	305	305
3	581.5	581.5	581.5
4	916	917	916.5

TABLE XV

Water Frequencies 28-inch Cylinder

AVARAGE	PERCENTAGE	PERCENTAGE	PERCENTAGE
202	202	202	1
222	222	222	2
1030	1030	1030	3

# TABLE IV

Water Production 20-1000 G/L/day

AVARAGE	PERCENTAGE	PERCENTAGE	PERCENTAGE
111	111	111	1
202	202	202	2
222	222	222	3
212	212	212	4

# TABLE IV

Water Production 20-1000 G/L/day

MODE	FREQUENCIES			AVERAGE
1	298	299.5	298.5	299
2	800	801	800	800
3	1457	1458	1457	1457

TABLE XVI

Air Frequencies 22-inch Cylinder

-----

MODE	FREQUENCIES			AVERAGE
1	171	171	172	171
2	465	466	465	465
3	854	855	855	855

TABLE XVII

Water Frequencies 22-inch Cylinder

DATE	DESCRIPTION	AMOUNT	CHECK NO.	BANK
1995	1000	1000	1000	1000
1995	1000	1000	1000	1000
1995	1000	1000	1000	1000

TABLE IV

Estimated 1995-1996

DATE	DESCRIPTION	AMOUNT	CHECK NO.	BANK
1995	1000	1000	1000	1000
1995	1000	1000	1000	1000
1995	1000	1000	1000	1000

TABLE V

Estimated 1995-1996



MODE	FREQUENCIES				AVERAGE
1	104	105	105	104	104.5
2	291	290	291	290	290.5
3	561.5	562	562	562	562
4	891	894	893	893	893
5	1278	1276	1276	1277	1277

TABLE XVIII

Air Frequencies 40-inch Cylinder

---

MODE	FREQUENCIES				AVERAGE
1	54.0	54.0	53.5	54.0	54.0
2	150.0	151.0	151	151	151
3	296	294	295	295	295
4	477	476.5	476	476	476
5	693	691	692	691.5	692

TABLE XIX

Water Frequencies 40-inch Cylinder

MODE	PERCENTAGE					AVERAGE
1	100	100	100	100	100	100
2	100	100	100	100	100	100
3	100	100	100	100	100	100
4	100	100	100	100	100	100
5	100	100	100	100	100	100

TABLE VIII

Air Propagation 10-inch Cylinder

MODE	PERCENTAGE					AVERAGE
1	24.0	24.0	24.0	24.0	24.0	24.0
2	150.0	150.0	150.0	150.0	150.0	150.0
3	200	200	200	200	200	200
4	100	100	100	100	100	100
5	600	600	600	600	600	600

TABLE IX

Water Propagation 10-inch Cylinder

MODE	FREQUENCIES			AVERAGE
1	131	131	131	131
2	353	351	352	352
3	659	658	658	658
4	1016	1020	1014	1017
5	1417	1419	1415	1417

TABLE XX

Air Frequencies 30-inch Cylinder  
with 6-inch Conical Ends

---

MODE	FREQUENCIES			AVERAGE
1	70	70	70	70
2	188	188	188	188
3	358	355	352	355
4	555	555	555	555
5	796	790	790	792

TABLE XXI

Water Frequencies 30-inch Cylinder  
with 6-inch Conical Ends

MODE	TESTING	TESTING	TESTING	MODE
1	101	101	101	101
2	202	202	202	202
3	303	303	303	303
4	404	404	404	404
5	505	505	505	505

# TESTING

Water pressure 30-40 psi  
with 6-inch nozzle

MODE	TESTING	TESTING	TESTING	MODE
1	101	101	101	101
2	202	202	202	202
3	303	303	303	303
4	404	404	404	404
5	505	505	505	505

# TESTING

Water pressure 30-40 psi  
with 6-inch nozzle

2. The following data was taken using the high speed movie camera.

MODE	FREQUENCIES		AVERAGE
1	61.7	61.7	61.7
2	170.5	170.5	170.5
3	332.0	332.0	332.0
4	539.5	539.5	539.5
5	787.5	787.5	787.5

TABLE XXII

Air Frequencies 52.7-inch Cylinder

-----

MODE	FREQUENCIES		AVERAGE
1			
2	86.2	86.2	86.2
3	169.1	169.1	169.1
4	279.0	279.0	279.0
5	419.0	419.0	419.0

TABLE XXIII

Water Frequencies 52.7-inch Cylinder

WAVELENGTH	RELATIVE INTENSITY	WAVELENGTH	RELATIVE INTENSITY
7.00	1.00	7.00	1.00
7.01	0.99	7.01	0.99
7.02	0.98	7.02	0.98
7.03	0.97	7.03	0.97
7.04	0.96	7.04	0.96

TABLE 1

RELATIVE INTENSITY OF 7.00-7.05 WAVELENGTHS

WAVELENGTH	RELATIVE INTENSITY	WAVELENGTH	RELATIVE INTENSITY
7.00	1.00	7.00	1.00
7.01	0.99	7.01	0.99
7.02	0.98	7.02	0.98
7.03	0.97	7.03	0.97
7.04	0.96	7.04	0.96

TABLE 2

RELATIVE INTENSITY OF 7.00-7.05 WAVELENGTHS

FIGURE VI  
PHOTOGRAPHS OF TYPICAL TWO NODED FREQUENCIES



TWO NODED WATER FREQUENCY



TWO NODED AIR FREQUENCY



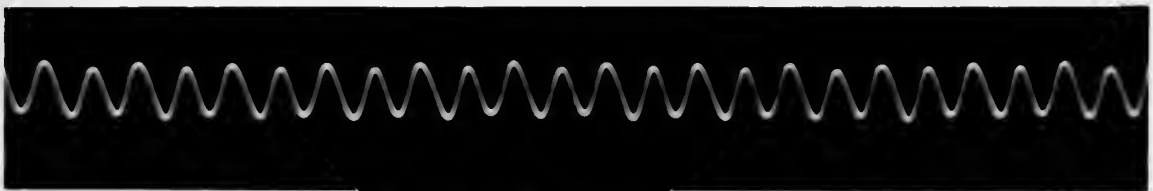


FIGURE VII

PHOTOGRAPHS OF TYPICAL SIX NODED FREQUENCIES



SIX NODED AIR FREQUENCY



SIX NODED WATER FREQUENCY



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